Estimating parameters of DC motors

White Paper

By Dipl.-Ing. Ingo Völlmecke

DC-motors are used in a wide variety of applications touching our daily lives, where they serve to relieve us of much work. However, the large number of DC-motors used is attended by a large amount of time and resources devoted to inspecting them at the end of their production cycle. The time needed for this process should be kept as brief as possible so that the inspection procedure is not the slowest part of the production process. Due to the increasing mass production of these motors, procedures for inspecting them have been developed which are able to determine the test objects' characteristic curves within seconds. Such procedures are known as Parameter Identification procedures. They determine the parameters without applying any external load, simply by measuring the current and voltage. The time and apparatus required for attaching a load and for aligning the test object with a loading mechanism can thus be totally omitted.

Introduction:

The dynamic behavior of DC motor can be described using two equations.

The first equation describes the electrical behavior:

$$u = R \cdot i + k \cdot \omega + L \cdot \frac{di}{dt} \qquad (1)$$

The second equation describes the mechanical behavior:

$$J\frac{d\omega}{dt} = k \cdot i - k_r \cdot \omega - M_L \qquad (2)$$

where the algebraic symbols represent the following:

Symbol	Unit	Definition
u	V	electric terminal voltage
i	A	electric armature current
ω	1/s	rotational freqency
R	Ω	Ohmic ferrule resistor
k	Vs	generator constant
L	H	inductivity
J	kgm²	moment of inertia
k_r	Nms	sliding friction
M_L	Nm	load

Note:

The load also reflects the moment of static friction inherent in the system.

Equations (1) and (2) can now be summarized in a single equivalent electromechanical circuit:

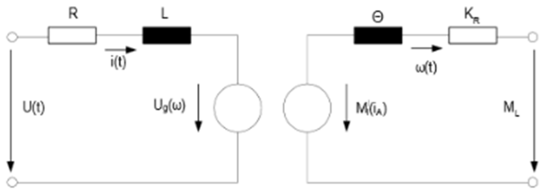

Figure 1: *Electromechanical equivalent circuit diagram*

Energy consideration:

For a better interpretation of the equations (1) and (2) an energy consideration is now performed.

Given by the equation (1) is multiplied with the current i and integrated over an indefinite interval.

$$\int u \cdot i dt = R\int i^2 dt + k\int \omega \cdot i dt + L\int i \frac{di}{dt} dt \qquad (3)$$

The first term in equation (3) describes the electrical energy supplied, the ohmic losses of the second, the third, the mechanical energy present in the system and the last one, the stored energy in the inductance.

The equation (2) is multiplied with the speed ω and integrated over an indefinite interval.

$$J\int \frac{d\omega}{dt} \cdot \omega dt = k\int i \cdot \omega dt - k_r\int \omega^2 dt - \int M_L \cdot w dt \qquad (4)$$

The first term in Equation (4) describes the rotational energy stored in the mechanical system, the second term, the mechanical energy, the third is the speed-proportional losses and the last the discharged mechanical energy loss as well as the energy contained therein due to the static friction.

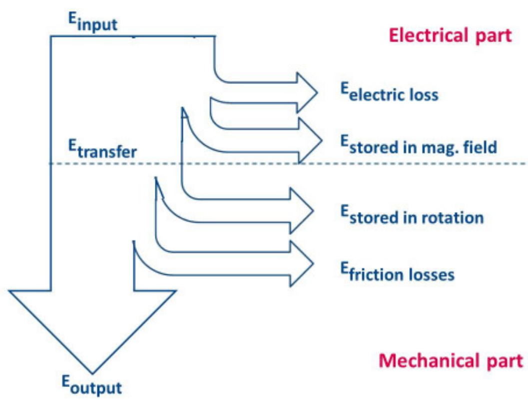

Figure 2: *Scheme of the energy distribution.*

Transfer function of a DC motor

Since only the terminal quantities voltage and armature current are used in estimating a DC motor's parameters, the course of the spectrum of the armature current-to-terminal voltage ratio is of particular interest. On the basis of this transfer function it is possible to make statements about at which point in the frequency range excitation is useful, since relevant parameter changes take effect in that frequency range.

A simple example should serve to affirm this:

To estimate the parameters of an electric low-pass with a cutoff frequency of 10 kHz, it isn't a practical approach to excite it with a 10 Hz oscillating quantity, since, considering the measurement precision, the input and output signals are approximately the same (tansfer factor approx. 1, phase-shift between input and output signals approx. 0 degrees). Only if the excitation approaches the cutoff frequency do the filter parameters become noticeable and is there a chance of determining them with reasonably high accuracy.

To determine the transfer function, Equations (1) and (2) are transferred into the Laplace domain.

$$U(s) = R \cdot I(s) + k \cdot \Omega(s) + L \cdot s \cdot I(s) \tag{5}$$

$$J \cdot s \cdot \Omega(s) = k \cdot I(s) - k_r \cdot \Omega(s) - M_L \tag{6}$$

From Equation (5) we thus find the rotational frequency:

$$\Omega(s) = \frac{k \cdot I(s) - M_L}{k_r + s \cdot J} \tag{7}$$

By using (7) in (5) and making some rearrangements, we obtain the equation:

$$U(s) = \frac{LJs^2 + s(RJ + Lkr) + RK_r + k^2}{k_r + sJ} I(s) - \frac{kM_L}{J} \frac{1}{\left(s + \frac{k_r}{J}\right)s} \tag{8}$$

Equation (8) describes the relationship between the terminal quantites U and I and to the load M_L.

From equation (8) thus follows:

$$\frac{I(s)}{U(s) + \frac{kM_L}{J}\frac{1}{\left(s + \frac{k_r}{J}\right)s}} = \frac{1}{L}\frac{s + \frac{k_r}{J}}{s^2 + s\left(\frac{R}{L} + \frac{k_r}{J}\right) + \frac{R}{L}\left(\frac{k_r}{J} + \frac{k^2}{RJ}\right)} \tag{9}$$

The following simplifications now follow:

Symbol	Unit	Definition
T_{ele}	s	Electrical time constant
T_{mech}	s	Mechanical time constant
K_A	1/s	Run-out constant
V	A/Vs	Amplification factor
$U^{\#}(s)$	V	$U(s) + \frac{kM_L}{J}\frac{1}{\left(s + \frac{k_r}{J}\right)s}$

With these abbreviations, the transfer equation follows:

$$\frac{I(s)}{U^{\#}(s)} = V \frac{s + k_A}{s^2 + s\left(\frac{1}{T_{ele}} + k_A\right) + \frac{1}{T_{ele}}\left(k_A + \frac{1}{T_{mech}}\right)} \tag{10}$$

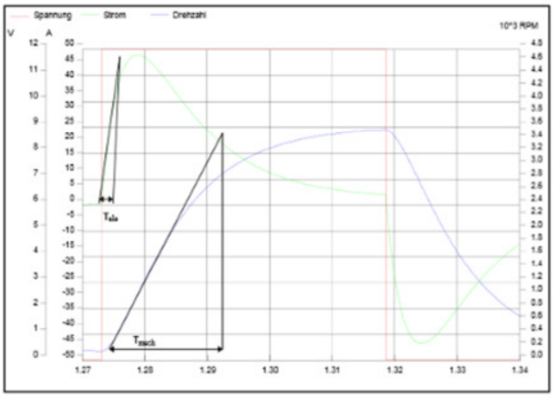

Figure 3: *Electrical and mechanical time constant of the DC electric motor.*

The constant T_{ele} is referred to as the electrical time constant of the DC machine. The electrical time constant is a measure of the response time of the current change in the terminal voltage.

The constant T_{mech} is referred to as the mechanical time constant of the DC motor. The mechanical time constant is a measure of the RPM's reaction time upon change in the terminal voltage.

Using Equation (9), the DC-motor's frequency response can be shown for fixed parameters.

The figures below show the frequency responses in terms of both magnitude and phase, as well as the characteristic curve for a motor with given parameters.

Parameter	Unit	Value
R	Ω	0.19
L	H	0.0005
k	Vs	0.0323
J	kgm^2	7.5e-5
kr	Nms	2e-5

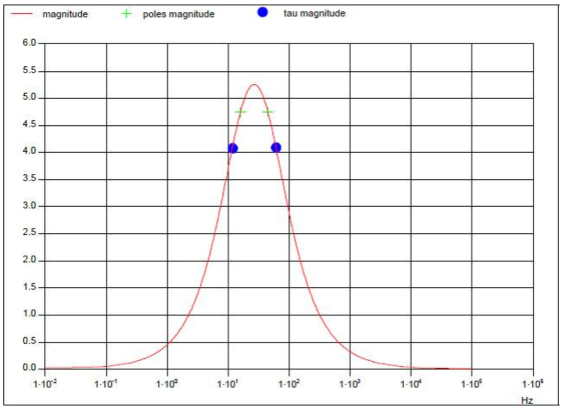

Figure 4: *Magnitude frequency response of a DC motor.*

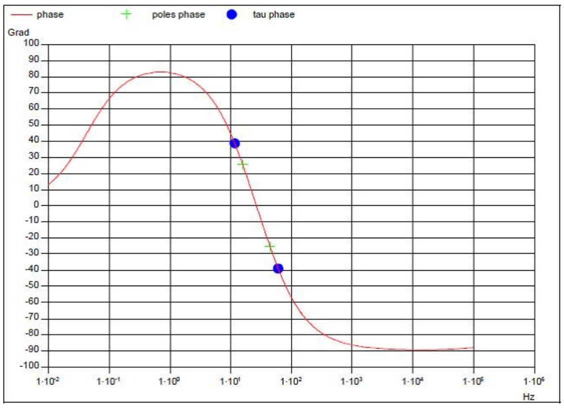

Figure 5: *Phase frequency response of a DC motor.*

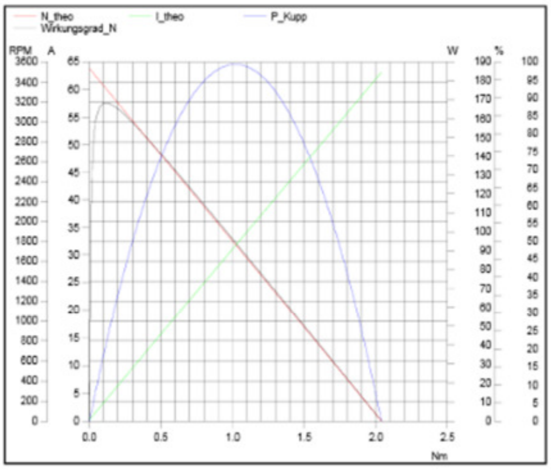

Figure 6: *Characteristic curve of a DC motor.*

In addition to the course of the respective frequency responses, the mechanical and electrical time constants are still proportional to frequencies, as well as the locations of the frequencies of the poles of the transfer functions.

The magnitude frequency response of the DC motor corresponds to a bandpass filter with a center frequency which lies between the two time constants of the motor. The phase frequency response of the DC otor corresponds to the phase frequency response of a bandpass filter. The zero in the numerator of the transfer function leads to the phase shift at low frequencies to zero.

The DC motor in the illustrated example has two real poles in the transfer function. These poles occur as a simple but separate poles in the frequency response. In contrast, there are also motors with complex conjugate poles, which can be represented in the frequency response as a double pole at the center frequency of the bandpass filter.

When considering the transfer function, spaces can now be specified in which the DC motor parameters can be estimated. As indicated above, a function of the zero point is obtained at the bottom of the transfer function in the numerator, but this zero point depends only on the expiry constant. Furthermore, in this region, the magnitude frequency response is near zero, so that there is no meaningful estimate that can be carried out and the discharge constant thus can not be determined with reasonable accuracy.

In the area of the time constant of the DC motor, sufficiently large amplitudes arise for the estimation of the poles of the system. From the poles, then, the DC motor parameters can be calculated.

From Equation (10), we see that for the zero-crossings:

$$s_{01,2} = \frac{\frac{1}{\tau_{ele}} + k_A}{2} \pm \sqrt{\left(\frac{\frac{1}{\tau_{ele}} + k_A}{2}\right)^2 - \frac{1}{\tau_{ele}}\left(k_A + \frac{1}{\tau_{mech}}\right)} \quad (11)$$

The system's poles become purely real if the square root expression is positive, from which follows:

$$\tau_{mech} > 4\frac{\tau_{ele}}{\left(1 - k_A \cdot \tau_{ele}\right)^2} \quad (12)$$

All DC motors meeting the conditions in Equation (12) have purely real poles.

Estimating the transfer function's poles:

For all motors for which Equation (12) applies, the following approach to determining the parameters is available:

$$\frac{I(s)}{U^{\#}(s)} = V\frac{s + k_A}{\left(1 - sT_1\right)\left(1 - sT_2\right)} \quad (13)$$

this equation can be multiplied out to yield:

$$\frac{I(s)}{U^{\#}(s)} = \frac{V}{T_1 T_2}\frac{s + k_A}{s^2 + s\frac{T_1 + T_2}{T_1 T_2} + \frac{1}{T_1 T_2}} \quad (14)$$

By comparing coefficients with Equation (9), we obtain the following conditional equations for the DC-motor's parameters:

$$L = \frac{T_1 T_1}{V} \quad (15)$$

$$R = \left(\frac{T_1 + T_2}{T_1 T_2} - k_A\right)L \quad (16)$$

$$\frac{k^2}{J} = \left(\frac{L}{T_1 T_2} - Rk_A\right) \quad (17)$$

If the poles of the denominator polynomial are determined and additionally the discharge constant of the motor is still known, the resulting parameters of the motor can be calculated. The real object of the estimation consists only in determining the poles of the denominator polynomial.

With a closer examination of the transfer function and of the frequency response, it will be appreciated that the system can be split into a high-pass and a low-pass.

$$\frac{I(s)}{U^{\#}(s)} = V\frac{s + \frac{k_r}{J}}{(1 + sT_1)(1 + sT_2)} = V\frac{s + \frac{k_r}{J}}{1 + sT_2}\frac{1}{1 + sT_1} \quad (18)$$

The first term in equation (18) represents a high-pass filter, the second term is a low-pass filter.

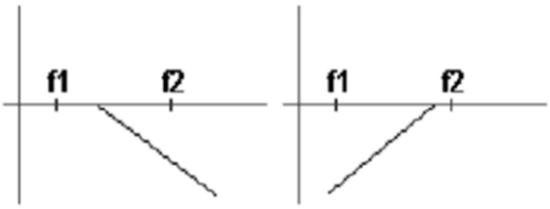

Figure 7: Representation of the individual frequency responses of the DC motor.

If the motor is now energized sequentially at two frequencies f_1 and f_2, the factors of the equation (18) can be separated and calculated separately.

As can be seen immediately in Figure 3, the low-pass yields at ω1 an excitation with an amount approximately at a factor of 1 and a phase shift of zero degrees, so that the low-pass filter can be neglected in a first approximation. Thus, the following transfer function:

$$\frac{I(\omega_1)}{U^{\#}(\omega_1)} \approx V_1 \frac{j\omega_1 + \frac{k_r}{J}}{1 + j\omega_1 T_2} \quad (19)$$

Now, multiplying the equation (19) out and separate them into a real and imaginary part, we obtain the system of equations (The indices I and R always denote the real and imaginary part of the respective size e.g., $U_R = \mathrm{Re}\,\{U^{\#}(j\omega)\}$.

$$\begin{bmatrix} I_R - \omega_1 T_2 I_I = V_1 U_R^* \\ I_I + w_1 I_R T_2 = V_1 U_I^* \end{bmatrix} \quad (20)$$

The following identities are used:

$$\begin{bmatrix} U_R^* = U_R \frac{k_r}{J} - \omega_1 U_I \\ U_I^* = U_I \frac{k_r}{J} + w_1 U_R \end{bmatrix} \quad (21)$$

and the equations for determining T_2 and V_1

$$\begin{bmatrix} T_2 = \dfrac{I_R U_I^* - I_I U_R^*}{w_1 (I_R U_R^* + I_I U_I^*)} \\ V_1 = \dfrac{I_R^2 + I_I^2}{U_R^* I_R + U_I^* I_I} \end{bmatrix} \quad (22)$$

Thus we obtain the first value estimate for determining a pole of the transfer function T_2 and of the gain factor V_1 .

These value estimates can now be used in estimating the second parameter. For this purpose, Equation (18) is rearranged as follows:

$$I(w_2)(1 + j\omega_2 T_1) = V_2 U(w_2) \frac{jw_2 + \frac{k_r}{J}}{1 + jw_2 T_2} \quad (23)$$

We now solve for T_1 and V_2, which, after separating the real and imaginary components in Equation (24), are determined by the conditional equations:

$$\begin{bmatrix} T_1 = \dfrac{I_R U_I^{\circ} - I_I U_R^{\circ}}{w_2 (I_R U_R^{\circ} + I_I U_I^{\circ})} \\ V_2 = \dfrac{I_R^2 + I_I^2}{U_R^{\circ} I_R + U_I^{\circ} I_I} \end{bmatrix} \quad (24)$$

using the following identities:

$$\begin{bmatrix} U_R^{\circ} = \dfrac{U_R \frac{k_r}{J} - U_I \omega_2 + \omega_2 T_2 (\omega_2 U_R + U_I \frac{k_r}{J})}{1 + \omega_2^2 T_2^2} \\ U_I^{\circ} = \dfrac{U_I \frac{k_r}{J} + U_R \omega_2 + \omega_2 T_2 (\omega_2 U_I - U_R \frac{k_r}{J})}{1 + \omega_2^2 T_2^2} \end{bmatrix} \quad (25)$$

Using Equations (22) and (24), the motor parameters can now be determined by iteration. Use the results from (22) in (24) until the iteration algorithm converges respectively. Then, the parameters of the test object can be determined using equations (15) to (17).

Estimating the parameters of a sample motor:

For the above example on a real motor, the following transfer function yields:

$$\frac{I(s)}{U^{\#}(s)} = \frac{1}{0.0005} \frac{s + \frac{2 \cdot 10^{-5}}{7.5 \cdot 10^{-5}}}{s^2 + s\left(\frac{0.19}{0.0005} + \frac{2 \cdot 10^{-5}}{7.5 \cdot 10^{-5}}\right) + \frac{0.19}{0.0005}\left(\frac{2 \cdot 10^{-5}}{7.5 \cdot 10^{-5}} + \frac{0.0323^2}{0.19 \cdot 7.5 \cdot 10^{-5}}\right)} \quad (26)$$

this results in:

$$\frac{I(s)}{U^{\#}(s)} = 2000 \frac{s + 0.2666}{s^2 + 380.2666s + 27922.4} \quad (27)$$

with the following characteristic parameters:

$$T_{ele} = 2.63ms$$
$$T_{mech} = 13.66ms$$
$$\frac{k_r}{J} = 0.2666$$

Typical current and voltage waveforms with the reciprocal of the electrical and mechanical time constants as excitation frequencies are shown in Figure 8.

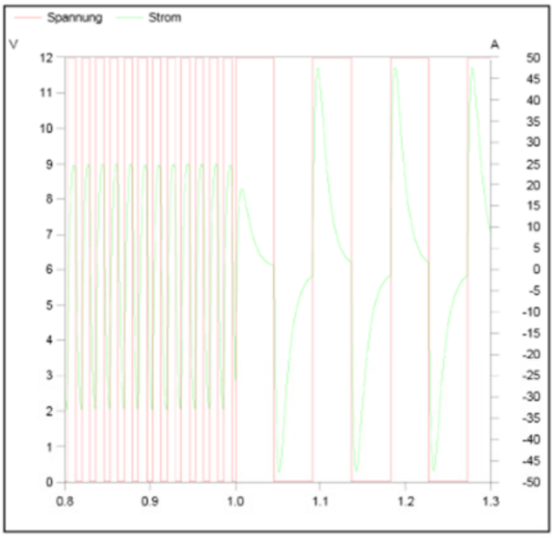

Figure 8: *Current and voltage curves for the determination of the parameters of the sample motor.*

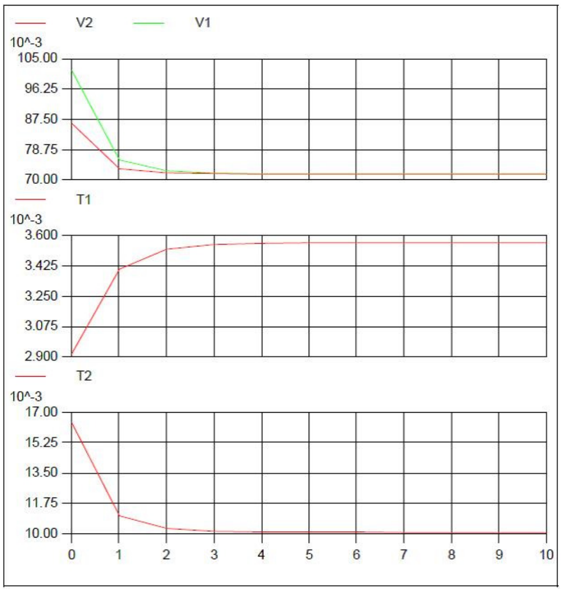

Figure 9: *Plot of estimated parameters T1, T2, V1, V2*

In figure 9, the convergence produced by this procedure is clear to see. The iteration ultimately returns the following parameters:

T1=0.00356063

T2=0.0100582

V1=0.0716225

V2=0.0716479

This translates to the following parameters for the test object:

R= 0.1904

L= 0.000501

k= 0.03233

Thus, the accuracy of the parameters determined is within 0.2%.

Additional information:

imc Test & Measurement GmbH
Voltastr. 5
13355 Berlin, Germany

Telephone: +49 (0)30-46 7090-0
Fax: +49 (0)30-46 31 576
E-mail: hotline@imc-tm.de
Internet: http://www.imc-tm.com

imc Test & Measurement GmbH is a manufacturer and solution provider of productive test and measurement systems. imc implements metrological solutions for research, development, service and production. imc has particular expertise in the design and production of turnkey electric motor test benches. Precisely outfitted sensor and telemetry systems complement our customer applications.

Our customers from the fields of automotive engineering, mechanical engineering, railway, aerospace and energy use imc measurement devices, software solutions and test stands to validate prototypes, optimize products, monitor processes and gain insights from measurement data. As a solution provider, imc offers their customers an attractive and comprehensive range of services. These include project consulting, contracted measurements, data evaluation, specialist deployment, customer-specific software development and system integration. imc consistently pursues its claim of providing services for "productive testing".

If you would like to find out more specific information about imc products or services in your particular location, or if you are interested in becoming an imc distributor yourself, please go to our website where you will find both a world-wide distributor list and more details about becoming an imc distributor yourself:

http://www.imc-tm.com/our-partners/