Measurement uncertainty of measured value series

Applications in imc FAMOS

Contents

This document describes how to deal with measurement uncertainty in connection with complete data records, for example, time-series or other series of measured values.

The GUM applies

Reference is made to the

GUM, **G**uide to the expression of **u**ncertainty in **m**easurement

and therein especially to:

JCGM 100:2008

The GUM applies. But it must be sensibly interpreted and applied.

When measurement uncertainty is mentioned in the following, the standard measurement uncertainty is meant.
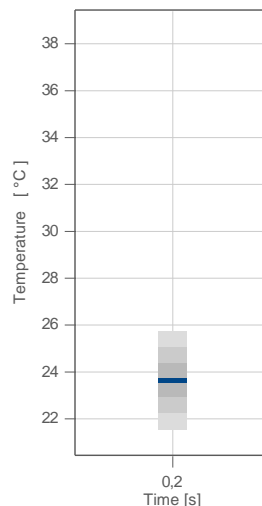
GUM and "the measured value"

The GUM deals with the measurement uncertainty for the one measured value. Example: The temperature of the oil in the oil sump is to be determined. A thermocouple and a measuring device is used for this. The measured value can be read off: 23.8 °C. This is the one measured value. The GUM describes how the measurement uncertainty of this measured value is now determined. To do this, all influencing factors and sources of error are listed and the measurement uncertainty budget determined. The inaccuracy of the thermocouple, the influence of the inaccurate reference point, the gain and the digitisation all play a role. In this example, let's assume the measurement uncertainty has been determined to be 0.7 °C.
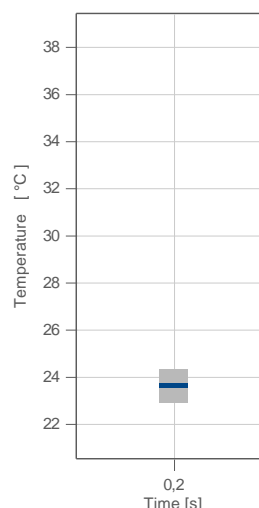
The interval is shown with a grey background in the illustration:

[Measured value - measurement uncertainty, measured value + measurement uncertainty]

It should be noted that the measurement uncertainty represents the standard deviation. With an underlying normal distribution of the measured values, the true value lies within the illustrated interval in only about 68 % of all measurements. In addition, the following illustration shows the range of the double and triple standard deviation, where the percentage rises to 95 % and 99.7 % respectively.

In the case of the entire interval comprising 6 standard deviations (6 Sigma), it can be said with very high certainty that the measured value really lies within this interval.

Repetition of the measurement to improve the measured value

The measured value is now read again. This time a value of 24.0 °C is obtained. The GUM describes how multiple measurements can reduce the measurement uncertainty. It is described under the heading "Type A evaluation". The measurement uncertainty – following the laws of standard deviation – decreases with $\sqrt{N}$, where N is the number of repetitions.

$$\bar{u} = \frac{u}{\sqrt{N}}$$

In the example, a measurement uncertainty of 0.5 °C is obtained if N=2. The resulting measured value is the arithmetic mean of the measured values and is calculated to be 23.9 °C.

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

Important here is the fact that the work takes place under the same measurement conditions. What is also important, however, is that the true value of the temperature has not changed.

Carrying out a further determination of the measured value

Here, too, the measured value is read again. But this time it is assumed that the true value of the temperature may already have changed. The intention is therefore not to improve the previous measured value, but to read a completely new measured value at this later time. According to GUM, a measurement is performed once again. However, it is a different, new measurement. Consequently, the measurement uncertainty budget must be determined for this measurement, too.

This gives rise to two cases, which are dealt with separately below: the constant and the non-constant measurement uncertainty budget.
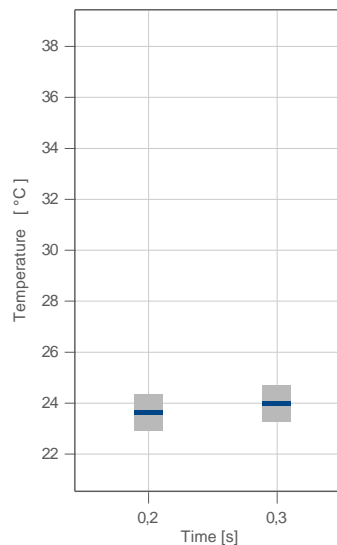
Constant measurement uncertainty budget

Here, it is assumed that – even during the determination of the measurement uncertainty budget – the measurement uncertainty will turn out to be dependent neither directly nor indirectly on the time. According to this assumption the measurement uncertainty budget doesn't even need to be determined.

An example of this is the temperature measurement mentioned above. The measurement uncertainty of the amplifier and the thermocouple used does not change. Both are read from the respective data sheet and are thus to be regarded as a constant.
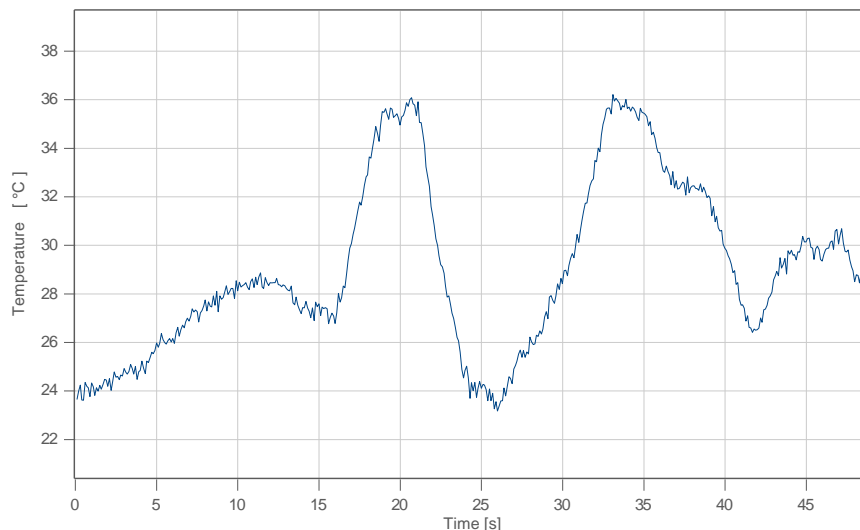
Measurement over time

With a constant measurement uncertainty budget, complete data records can now be acquired. The measurement uncertainty applies to every single measuring point and thus to all of them.


First measurement at 0.2s, the next at 0.3s.

The complete data record looks like this:

Here, therefore, the entire measurement (= data record) can be characterised with a single value for the measurement uncertainty.

The measurement uncertainty is shown as a band with a dark grey background. The band has a height of ±0.7 °C everywhere, even if it doesn't seem like it at first glance in the steep places.
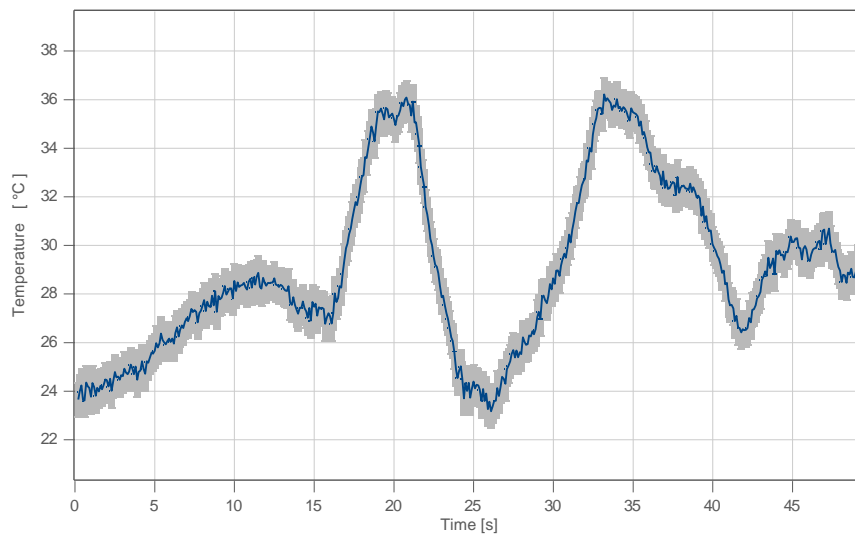
Here, too, the ranges of the double and triple standard deviation are shown with a brighter background:
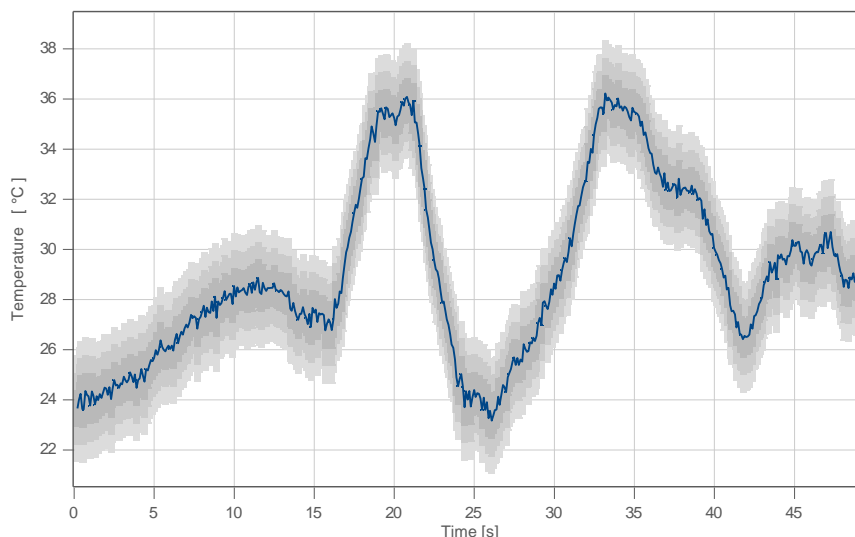
If the measurement uncertainty is constant, this simplifies the further processing based on the data records. Thus, the measuring channel in imc FAMOS is assigned a fixed measurement uncertainty that can then be used conveniently for further calculation.

Non-constant measurement uncertainty budget

If a different value is obtained for the measurement uncertainty in the new measurement – other than 0.7 °C in the example above – then that is the general case of the GUM: the measurement uncertainty budget must be determined for each measurement, i.e. for each measured value. A simplified treatment is not possible. Typical examples are:

- A measured value is read every day. Decisive influencing factors are temperature and air pressure, which can be significantly different every day.
- The measurement uncertainty strongly depends of the size of the measured value itself, e.g. amplitude-proportional.

Example: force measurement with measurement uncertainty dependent on the measured value

The example of a force measurement with an amplifier is discussed below. The data sheet for the amplifier provides the following information:

Zero point deviation typically 1 % of the measuring range, maximum 2.5 %

Gain uncertainty typically 1.5 % of the measured value, maximum 4 %

Measuring range 200 N

The values are interpreted such that the contribution to the measurement uncertainty caused by zero point deviation is equivalent to the typical value, i.e. 2 N. In addition, it is assumed that the amplitude-proportional share is also equivalent to the typical value, i.e. 3 N at maximum output. The force-dependent measurement uncertainty can be determined using this equation:

Measurement uncertainty (force) = 2 N + force * 0.015.

The curve of the force is shown in the following illustration:
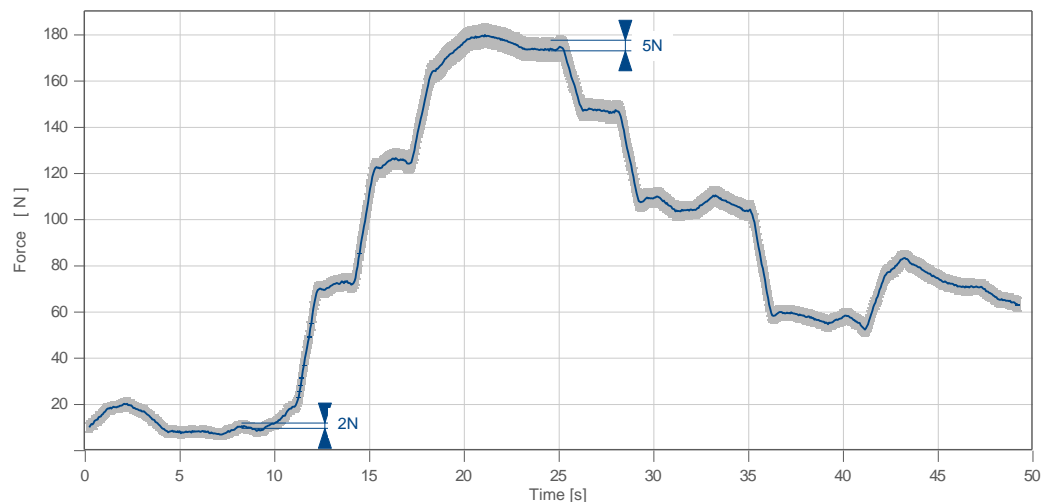
In the illustration the interval [measured value - measurement uncertainty, measured value + measurement uncertainty] has a grey background. With small forces (around 0 N), a measurement uncertainty of 2 N is obtained. With large forces (around 200 N), a measurement uncertainty of (2 + 3) N = 5 N is obtained.

The two envelopes can be generated with the following imc FAMOS commands:

```
tol1 = Force + 2 + Force * 0.015
tol2 = Force - 2 - Force * 0.015
```
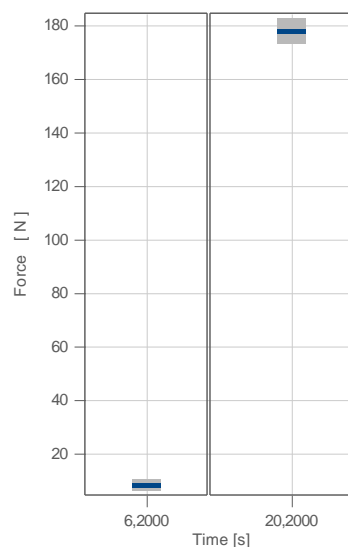
It should be noted that the amplifier has intentionally been given poor technical data in order to make the graphics clear. Modern amplifiers such as those from the imc CRONOS*flex* series exhibit values that are far more than 10 times as good.

Consequences

If the result of a measurement or an evaluation is to be illustrated, then the measurement uncertainty is always to be stated as well. If the measurement uncertainty is a (single) numerical value, it is easy to illustrate, to document and to be recorded and interpreted by the observer.

A measurement uncertainty that cannot be expressed as a numerical value is neither easy to handle nor easy for the observer to interpret or estimate.

Therefore a non-constant measurement uncertainty is frequently converted to a constant one.

Formation of the maximum value

For example, a non-constant measurement uncertainty can be converted into a constant one by forming a maximum value.
In the above example of force measurement, therefore

[constant measurement uncertainty] = maximum (measurement uncertainty (force)) = 5 N

The "worst case" principle is applied here: if in doubt, the greater or worse value is taken. Caution: the consistent application of this principle, if necessary through several stages of evaluation, sometimes leads to very high and even unbelievable values!
The principle of maximum value formation does not exist in the GUM. That was only used in former times in the classic error calculation: maximum errors were determined and their propagation calculated. The GUM prefers to make use of more reproducible variables such as the standard deviation. In addition, the principle of mutual cancellation is observed.


Comprehensive measurement uncertainty

An averaging operator is preferable when reducing a complex statement to a numeric value. The maximum value former takes into account only the maximum, i.e. a single value. The calculation of the standard deviation of all the values in the data record is preferable. This is the comprehensive standard deviation with which the comprehensive measurement uncertainty is defined. The equation known from the descriptive statistics

$$s = \sqrt{\frac{1}{G-1}\left[\sum_{i=1}^{N}\left((n_i - 1)s_i^2\right) + \sum_{i=1}^{N} n_i\,(\bar{x}_i - \bar{x})^2\right]}$$

where

s	comprehensive standard deviation
N	number of random samples = number of measured values
n_i	size of the random sample
G	total number, sum of all n_i
s_i	standard deviation of the random sample = measurement uncertainty of every measured value
$\bar{x}_i$	mean value of the random sample
$\bar{x}$	total mean value of all G values

is applied by declaring all mean values to be irrelevant: One imagines all measured values adjusted to zero, which is irrelevant for the consideration of the measurement uncertainty, i.e. $\bar{x}_i = 0$ and $\bar{x} = 0$. The mathematical background is that the subtraction of a fixed number (here of the measured value) from a random variable doesn't change its variance or standard deviation.
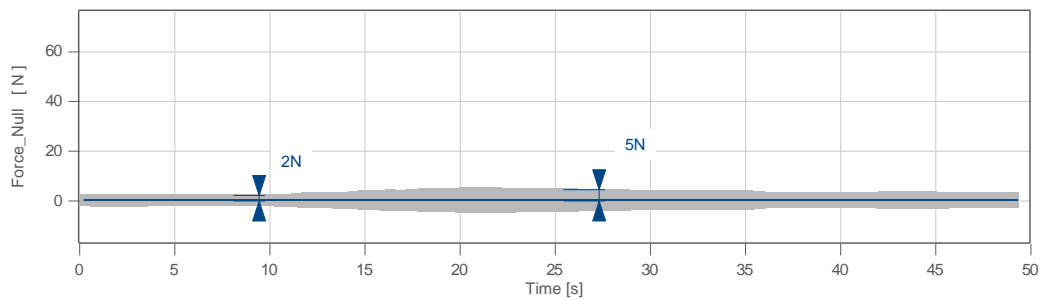
All measured values adjusted to zero, representation of the interval [±measurement uncertainty]

Since no random sample size is known for the standard deviations s_i of the individual measured values, all n_i are assumed to be equal and very large. Hence $G = N \cdot n_i$ and, as n_i approaches infinity, $n_i \rightarrow \infty$, this produces:

$$s = \sqrt{\frac{1}{N}\sum_{i=1}^{N} s_i^2}$$

The comprehensive standard deviation is represented as the quadratic mean value of all individual standard deviations.

The comprehensive measurement uncertainty u is thus given by

$$u = \sqrt{\frac{1}{N}\sum_{i=1}^{N} u_i^2}$$

where u_i = measurement uncertainty of the measured value and N = number of measured values

In the example of force measurement this results in:

u = 3.4 N

The statement of the comprehensive measurement uncertainty is this: The data record is considered a random process. The comprehensive measurement uncertainty describes all occurring deviations from the respective true value in the form of the standard deviation. This applies to any distribution, not just the normal distribution.

The following statement of the GUM therefore also applies: The measurement uncertainty states that, with an underlying normal distribution, the true value lies within the interval [±measurement uncertainty] around the measured value in 68 % of all measurements. Very similarly, the comprehensive measurement uncertainty states that, with an underlying normal distribution, the true value lies within the interval [±measurement uncertainty] around the respective measured value in 68 % of all measured values in the data record. The comprehensive measurement uncertainty is also just a measurement uncertainty that happens to characterise the entire data record.

Of course, this comprehensive value is not correct or not precise for each individual measured value. If one only knows the comprehensive measurement uncertainty, one has no knowledge of the individually different measurement uncertainties. Only those who have that knowledge can, for example, say the following: at the points with a particularly low individual measurement uncertainty, the true measured value lies with virtual certainty within the much broader tolerance band of the comprehensive measurement uncertainty. At the points with a particularly high individual measurement uncertainty, the true measured value lies with less probability (and probably doesn't lie) within the tolerance band of the comprehensive measurement uncertainty, which is then comparatively narrow.
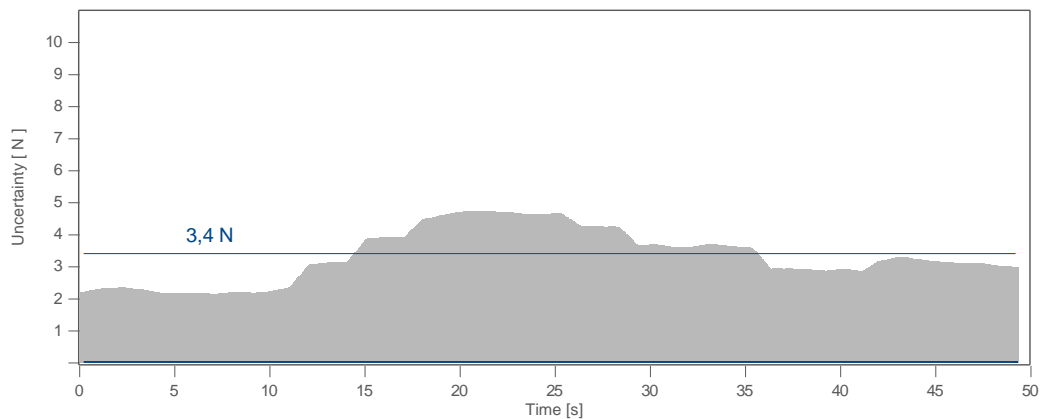
Shown here is the detailed illustration of the individual measurement uncertainty of the force curve and the superimposed line of the comprehensive measurement uncertainty. It should be noted here that 68 % of the measured values do not lie below the 3.4 N line. This is because the individual measurement uncertainties (i.e. the densely clustered grey bars) are not normally distributed.

Important: The comprehensive measurement uncertainty may be applied only if a complete data record can be really usefully characterised with a numerical value from the application.

imc FAMOS offers this comprehensive measurement uncertainty as a result of the propagation of the measurement uncertainty by a mathematical algorithm. It is up to the user to observe the value.

Interval formation

In all cases where the situation is complex, the measurement uncertainty not constant at all and a summary consideration too coarse, there is no choice but to subdivide the data record into sections. Each section is then regarded individually.
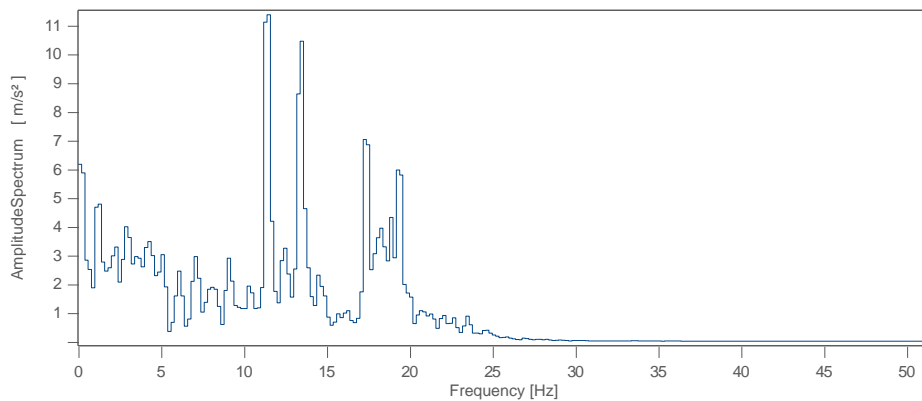
Complex example

An acceleration time curve "acc" is given.
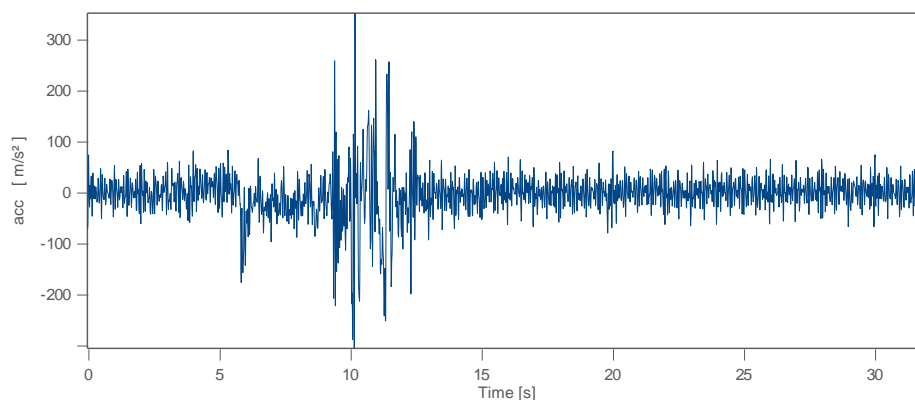
The amplitude spectrum is calculated with the following command:

```
AmplitudeSpectrum = AmpSpectrumRMS_1( acc, 500, 2, 0, 1)
```

The measurement uncertainty (0.5 % of the measuring range) of the signal "acc" is known and is 3 m/s².

The measurement uncertainty of the amplitude spectrum is determined by imc FAMOS:
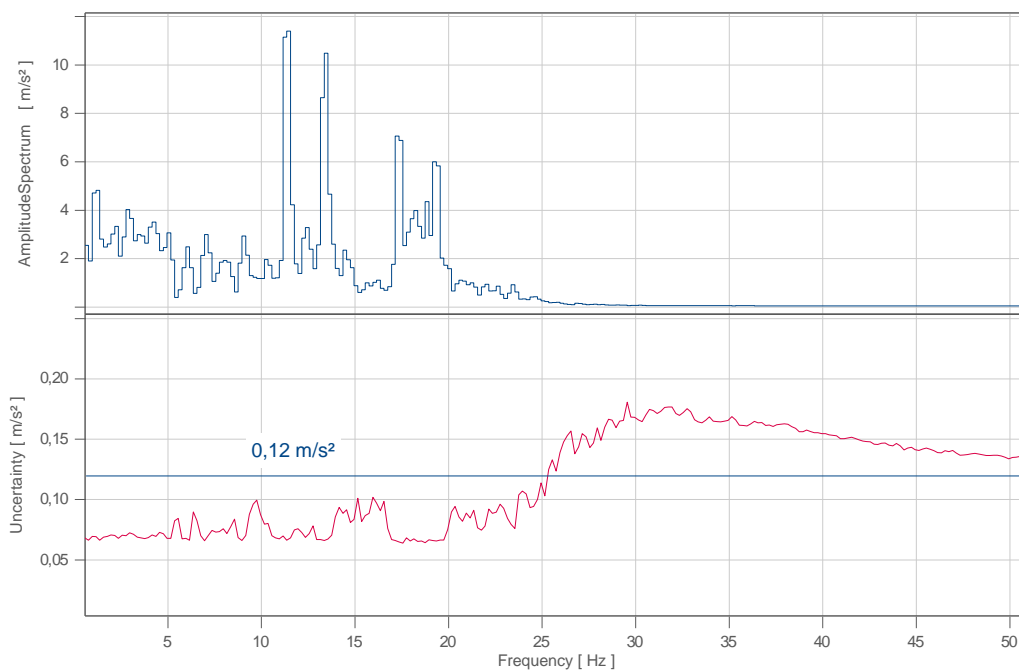
```
UncertaintySet( acc, "Uncertainty", 3)
UNCERTAINTY_LOOP 1000 1
     _acc = UncertaintyModify ( acc )
     AmplitudeSpectrum = AmpSpectrumRMS_1( _acc, 500, 2, 0, 1)
     UncertaintyCalc ( AmplitudeSpectrum )
End
uc = UncertaintyGet( AmplitudeSpectrum, "Uncertainty")
```

The result of the calculation is uc = 0.12 m/s².

The alternative command

```
     UncertaintyCalc ( AmplitudeSpectrum, 99, 0, "uc")
```

provides detailed information about the measurement uncertainty of each individual spectral line, i.e. each individual point of the result data record.

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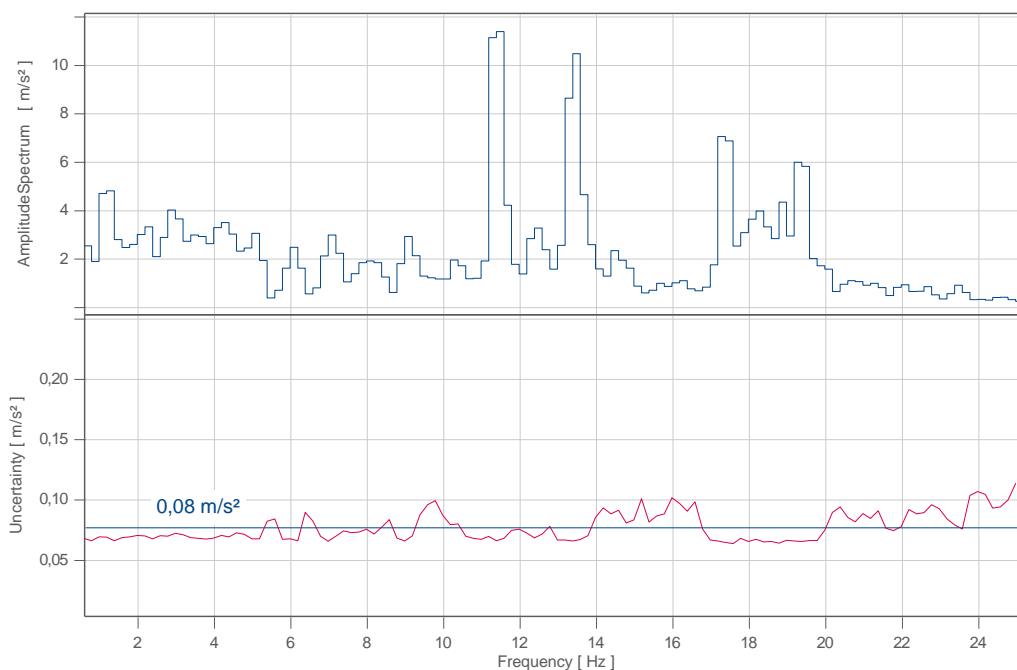
The detailed curve of the measurement uncertainty over the frequency shows that the measurement uncertainty is nothing like constant. The application decides whether the comprehensive measurement uncertainty of 0.12 m/s² may be used. For example, if only the largest spectral line is of interest, then a measurement uncertainty of 0.07 m/s² can be safely specified for it. But then it would be perfect if the mathematical algorithm were to immediately extract this line and determine the measurement uncertainty for precisely this line.

After all, it's good to know whether there are areas in the result with a conspicuously high measurement uncertainty. A detailed analysis of the individual measurement uncertainty is therefore always recommended. In the case of the spectral analysis carried out, the measurement uncertainty is high in the actually uninteresting right-hand part of the spectrum. If this part is cut out, the comprehensive measurement uncertainty also becomes significantly smaller in the remaining left-hand part. After addition to the algorithm of

```
AmplitudeSpectrum = cut ( AmplitudeSpectrum, 0, 25 )
```

it follows that u_c = 0.08 m/s². This can be now considered extremely representative of the result.

Conclusion: Detailed knowledge of the measurement uncertainty of each measurement point is important in order to justify summarisation to form a comprehensive measurement uncertainty.

Additional information:

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